

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 1 Round 1
Arithmetic: Percents

1.) _____ 60 _____

2.) _____ 6 _____

3.) _____ 13, 26 _____

1) A number N is increased by 20% and 5 is added to the result. If this sum is decreased by 30%, the result is 53.9. Find N .

$$0.7(1.2N + 5) = 53.9$$

$$0.84N + 3.5 = 53.9$$

$$0.84N = 50.4$$

$$N = 60$$

2) 60% of $(X+9)$ is equal to $(10X+3)\%$ of $\frac{100}{7}$. Find X .

$$0.6(X+9) = ((10X+3)/100)(100/7)$$

$$0.6(X+9) = (10X+3)/7$$

$$0.6X + 5.4 = (10X+3)/7$$

$$4.2X + 37.8 = 10X + 3$$

$$34.8 = 5.8X$$

$$X = 6$$

3) 20% of the students who applied to State U. from Rufus High School were accepted. 26% of the students who applied to State U. from Buford High School were accepted. If there were 150 students in total who applied from the two high schools, $Y\%$ of the total number of students who applied were accepted, and Y is an integer, what are all possible values for the number of students who were accepted to State U. from Buford High School.

Let x = # of students who applied from Rufus HS, z = # of students who applied from Buford HS. $0.2x + 0.26z = 0.01Y(150) = 1.5Y$, and $x+z=150$. Solve $x+z=150$ for x to get $x=150-z$, and substitute into the first equation to get $0.2(150-z) + 0.26z = 1.5Y$, so $30-0.06z=1.5Y$.

We know Y must be between 20 and 26. Both Y and z must be integers, and 26% of z must also be an integer. If $Y=21$, $0.06z=1.5$, so z is 25, but $0.26(25)$ is not an integer. . If $Y=22$, $0.06z=3$, so $z=50$, and $0.26(50)=13$. OK. If $Y=23$, $0.06z=4.5$, and $z=75$, but $0.26(75)$ is not an integer. If $Y=24$, $0.06z=6$, so $z=100$, and $0.26(100)=26$. OK. If $Y=25$, $0.06z=7.5$, and $z=125$, but $0.26(125)$ is not an integer. So 13 or 26 are the possible values.

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Match 1 Round 2 Algebra I: Equations

1.) _____-9_____

2.) _____1_____

3.) _____10, 45_____

1.) Solve for a:

$$3 - 2(a - 4(a - 1)) = 1 - 4(a - 2(a - 3))$$

$$3 - 2(a - 4a + 4) = 1 - 4(a - 2a + 6)$$

$$3 - 2(-3a + 4) = 1 - 4(-a + 6)$$

$$3 + 6a - 8 = 1 + 4a - 24$$

$$6a - 5 = 4a - 23$$

$$2a = -18$$

$$a = -9$$

2) Solve for b:

$$(2b + 5)(3b - 4) - (b + 4)(5b - 2) = (b + 10)(b - 3)$$

$$(6b^2 + 7b - 20) - (5b^2 + 18b - 8) = b^2 + 7b - 30$$

$$b^2 - 11b - 12 = b^2 + 7b - 30$$

$$-11b - 12 = 7b - 30$$

$$18 = 18b$$

$$b = 1$$

3) Solve for c:

$$0.3c - \frac{c + 6}{4} = \frac{c - 15}{c - 5}$$

$$4(c - 5)0.3c - (c - 5)(c + 6) = 4(c - 15)$$

$$1.2c^2 - 6c - (c^2 + c - 30) = 4c - 60$$

$$0.2c^2 - 7c + 30 = 4c - 60$$

$$0.2c^2 - 11c + 90 = 0$$

$$c^2 - 55c + 450 = 0$$

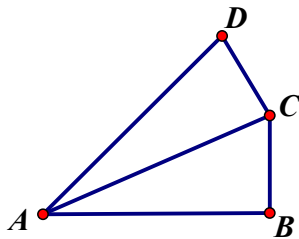
$$(c - 10)(c - 45) = 0$$

$$c = 10, c = 45$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

- 1.) $\underline{\hspace{1cm}} 3\sqrt{3} + 3\sqrt{5} + 6 \underline{\hspace{1cm}}$ cm
- 2.) $\underline{\hspace{1cm}} 16\sqrt{3} \underline{\hspace{1cm}}$ cm
3. $\underline{\hspace{1cm}} 54$ cm or 56 cm $\underline{\hspace{1cm}}$

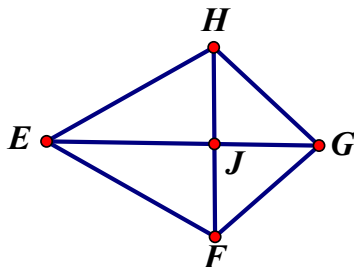


- 1) If angles ABC and ACD are right angles, $AC=6$ cm, and $BC=CD=3$ cm, find the perimeter of quadrilateral ABCD.

$$AB = \sqrt{(AC^2 - BC^2)} = \sqrt{6^2 - 3^2} = 3\sqrt{3} \quad \sqrt{6^2 + 3^2} = 3\sqrt{5} \quad AD = \sqrt{(AC^2 + CD^2)} = \sqrt{6^2 + 3^2} = 3\sqrt{5} \quad \text{Perimeter is } 3\sqrt{3} + 3\sqrt{5} + 6$$

- 2.) A rhombus with side length 4 cm has one angle of 60 degrees. What is the product of the diagonals of the rhombus?

One diagonal of the rhombus splits the rhombus into 2 equilateral triangles of side 4. The area of each is $\frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$, so the total area is $16\sqrt{3}$. The total area is half the product of the diagonals, so the product of the diagonals is $16\sqrt{3}$



3. The area is 168 cm^2 , so the diagonals must multiply to 336. There must be 4 integers that multiply to 336, 2 of which are identical, since one diagonal bisects the other.

$336=14*24$ and is also equal to $16*21$. Using $16*21$, the 2 identical sides could be 8 and 8, and those would be part of Pythagorean triples 6-8-10 and 8-15-17, and $6+15=21$, so you would have 2 triangles with hypotenuse 10 and 2 triangles with hypotenuse 17, which gives a solution of 54 cm. Using $14*24$, the 2 identical sides could be 12 and 12, and those would be part of Pythagorean triples 5-12-13 and 9-12-15 and $5+9=14$, so you would have 2 triangles with hypotenuse 13 and 2 triangles with hypotenuse 15, which gives a solution of 56 cm.

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Match 1 Round 4
Algebra 2:
Simultaneous Equations

1.) $\underline{\hspace{1cm}} a = \underline{4} \quad b = \underline{10}$

2.) $\underline{\hspace{1cm}} c = \underline{-6} \quad d = \underline{-4}$

3.) $\underline{\hspace{1cm}} k = \underline{-4}$

1.) Solve for a and b:

$$1.25a - 0.4b = 1$$

$$0.75a + 0.01b = 3.1$$

Multiply first equation by 40, second equation by 10

$$50a - 16b = 40$$

$$75a + b = 310$$

Multiply first equation by 3 and second equation by -2

$$150a - 48b = 120$$

$$-150a - 2b = -620$$

add these together to get $-50b = -500$, so $b=10$. $1.25a-4=1$, so $1.25a=5$, so $a=4$.

2.) Solve for c and d:

$$\frac{2}{c-4} + \frac{4}{d+3} = \frac{-21}{5} \quad \text{Let } u = \frac{1}{c-4}, v = \frac{1}{d+3} \quad 2u + 4v = \frac{-21}{5} \quad 5u - 6v = \frac{11}{2}$$
$$\frac{5}{c-4} - \frac{6}{d+3} = \frac{11}{2}$$

Multiply by 5 and 2

$$10u + 20v = -21$$

$$10u - 12v = 11$$

Subtract

$$32v = -32$$

$$v = -1, \text{ so}$$

$$-1 = \frac{1}{d+3}, d = -4$$

$$10u - 12(-1) = 11,$$

$$10u + 12 = 11,$$

$$10u = -1, u = \frac{-1}{10}$$

$$\frac{-1}{10} = \frac{1}{c-4}$$

$$c = -6$$

$$10u - 12(-1) = 11,$$

$$10u + 12 = 11,$$

$$10u = -1, u = \frac{-1}{10}$$

$$\frac{-1}{10} = \frac{1}{c-4}$$

$$c = -6$$

3) Find all values of k such that the system below has infinitely many solutions (x,y,z):

$$-2kx + 3y - z = 6$$

$$3x + ky + 2z = 13$$

$$12x + 25y - 11z = -34$$

Eliminate z

$$-4kx + 6y - 2z = 12$$

$$3x + ky + 2z = 13$$

$$(-4k + 3)x + (6 + k)y = 25$$

and

$$33x + 11ky + 22z = 143$$

$$24x + 50y - 22z = -68$$

$$57x + (11k + 50)y = 75$$

Since $3 \cdot 25 = 75$, We need

$$3(6 + k) = (11k + 50), 3(-4k + 3) = 57$$

$$18 + 3k = 11k + 50, -4k + 3 = 19$$

$$-8k = 32, -4k = 16, so$$

$$k = -4$$

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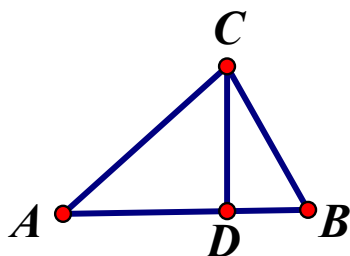
Match 1 Round 5
Trig: Right Triangles

1.) $\frac{12 + 4\sqrt{3} + 4\sqrt{2}}{10}$

2.) $\frac{2\sqrt{39}}{13}$

3.) $\frac{24\sqrt{5} - 15\sqrt{2}}{10}$

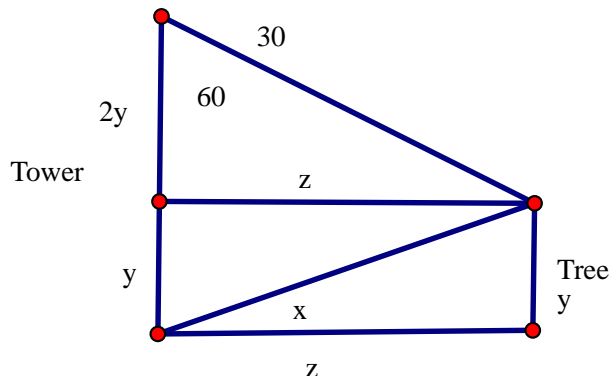
1.), For $\triangle ABC$, an altitude is drawn from C and intersects \overline{AB} at point D . If $\tan(\angle CBA) = 1$ and $\sin(\angle CAB) = 0.5$ and $CD = 4$, what is the perimeter of $\triangle ABC$?



$\tan(\angle CBA) = 1$, and $CD=4$, so $BD=4$ and $BC=4\sqrt{2}$.

$\sin(\angle CAB) = 0.5$, so since $CD=4$, $AC=8$, and $AD=4\sqrt{3}$. The sum of the sides of $\triangle ABC$ is

2) A tree is located a certain horizontal distance from an observation tower. When viewed from the top of the tower, the angle of depression to the top of the tree is 30 degrees. When viewed from the bottom of the tower, the angle of elevation to the top of the tree is x degrees. If the tower is 3 times as tall as the tree, what is $\cos(x)$?



$$\tan(60) = \frac{z}{2y}$$

$$z = y(2\sqrt{3})$$

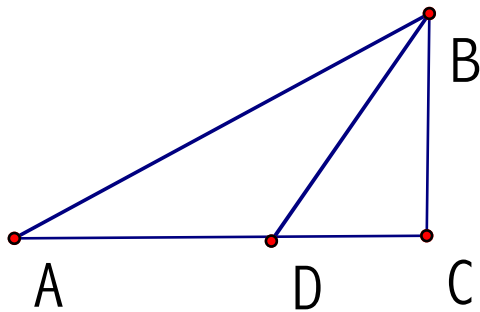
The length of the line from the bottom of the tower to the top of the tree is

$$\sqrt{y^2 + (2y\sqrt{3})^2} = y\sqrt{13}$$

$$\cos(x) = \frac{2y\sqrt{3}}{y\sqrt{13}} = \frac{2\sqrt{39}}{13}$$

3.) In triangle ABC below, $\angle C$ is a right angle, $\cos(\angle BAC) = \frac{2}{3}$,

$\cos(\angle BDC) = \frac{1}{3}$, and $BC = 6$. What is the length of AD?



$$\cos(\angle BDC) = \frac{1}{3}, \sin(\angle BDC) = \frac{2\sqrt{2}}{3}, \tan(\angle BDC) = 2\sqrt{2}, \text{ so}$$

$$2\sqrt{2} = \frac{BC}{CD} = \frac{6}{CD} \setminus CD = \frac{6}{2\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$\cos(\angle BAC) = \frac{2}{3}, \sin(\angle BAC) = \frac{\sqrt{5}}{3}, \tan(\angle BAC) = \frac{\sqrt{5}}{2}$$

$$\cos(\angle BAC) = \frac{2}{3}, \sin(\angle BAC) = \frac{\sqrt{5}}{3}, \tan(\angle BAC) = \frac{\sqrt{5}}{2}$$

$$\frac{\sqrt{5}}{2} = \frac{BC}{AC} = \frac{6}{AC} \setminus AC = \frac{12}{\sqrt{5}} = \frac{12\sqrt{5}}{5}$$

$$AD = \frac{12\sqrt{5}}{5} - \frac{3\sqrt{2}}{2} = \frac{24\sqrt{5}}{10} - \frac{15\sqrt{2}}{10} = \frac{24\sqrt{5} - 15\sqrt{2}}{10}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 1 Round 6
Coordinate Geometry

1. _____ $\frac{13}{4}$ _____

2.) _____ $2\sqrt{17}$ _____

3.) _____ $\frac{-13}{6}$ _____

- 1.) Point A has coordinates (-5, -3), point B has coordinates (3,-6), and point C has coordinates (1,2). A median of $\triangle ABC$ is drawn from point C to side \overline{AB} . Find the slope of the line containing the median.

The midpoint of segment AB is (-1,-4.5). The slope is then $\frac{2 - (-4.5)}{1 - (-1)} = \frac{6.5}{2} = \frac{13}{4}$

- 2.) $2x-3y=6$ bisects the segment with endpoints (a,5) and (8,7). Find the length of the segment.

The y-coordinate of the midpoint is 6, and must solve $2x-3y=6$, so if $2x-3(6)=6$, then $2x=24$ and $x=12$. If the midpoint has x-coordinate 12, then a must be 16. The length

is then $\sqrt{(16-8)^2 + (7-5)^2} = \sqrt{68} = 2\sqrt{17}$

- 3.) An isosceles trapezoid has its longer base with endpoints (-5,7) and (4,-5). The shorter base is $\frac{2}{3}$ of the length of the longer base. One of the endpoints of the

shorter base has coordinates (k,6). Find k. The longer base has slope $\frac{-4}{3}$ and

length $\sqrt{(4 - (-5))^2 + (-5 - 7)^2} = 15$, so the shorter base has length 10. Since the trapezoid is isosceles, the shorter base forms one side of a rectangle with the middle 10 units of the longer base. If you cut the longer base into 6 equal parts, the middle 10 units, go from (-3.5, 5) to (2.5,-3). The slopes from each of those

points to the longer base must be $\frac{3}{4}$, so the x-coordinate of the point with y-

coordinate 6 must solve $6-5 = \frac{3}{4}(x+3.5)$, so $1 = \frac{3}{4}x + \frac{21}{8}$, so $x = \frac{\frac{-13}{8}}{\frac{3}{4}} = \frac{-13}{6}$.

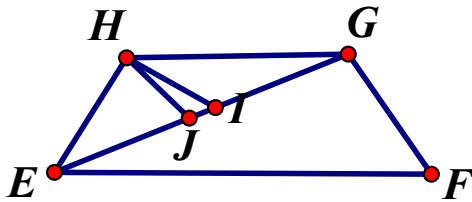
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-15 Match 1 Team Round

1.) $\frac{15\sqrt{10} - 20\sqrt{5}}{2}$ 4.) $\frac{7}{3}, \frac{9 \pm \sqrt{13}}{2}$

2.) $\frac{3\sqrt{5}}{5}$ 5.) $25, -12$

3.) $x = 200, y = 10, z = 50$ 6.) 20

1)



1. Isosceles trapezoid EFGH has $EH = 5\sqrt{2}$ and angle HEF measures 45 degrees. The area of the trapezoid is 75 cm^2 . Diagonal \overline{EG} measures $5\sqrt{10} \text{ cm}$. The median of $\triangle EGH$ from angle H is drawn to point I on \overline{EG} . The angle bisector of angle EHG intersects \overline{EG} at point J. What is the length of segment \overline{IJ} ?

Since angle HEF is 45 degrees and $EH = 5\sqrt{2}$, each of the triangles on either side of the trapezoid must be isosceles right triangles of side 5, so they each have area 12.5 cm^2 , so a total of 25 cm^2 . That means GH must be 10, since the height of the trapezoid is 5, and the middle rectangle of the trapezoid must give the other 50 cm^2 . The distance EI is

$\frac{5\sqrt{10}}{2}$ by definition of median. The angle bisector of angle EHG splits GE into the

same ratio as GH to HE, so let $x = EJ$.

$$\frac{5\sqrt{2}}{10} = \frac{x}{5\sqrt{10} - x}$$

Then $10x = 25\sqrt{20} - 5x\sqrt{2}$, so $10x = 25\sqrt{20} - 5x\sqrt{2}$, and

$$x = \frac{50\sqrt{5}}{10 + 5\sqrt{2}}$$

Subtract this from (1/2) the length of the diagonal $\frac{5\sqrt{10}}{2}$

$$\begin{aligned} & \frac{5\sqrt{10}}{2} - \frac{50\sqrt{5}}{10 + 5\sqrt{2}} = \\ & \frac{5\sqrt{10}}{2} - \frac{50\sqrt{5}(10 - 5\sqrt{2})}{(10 + 5\sqrt{2})(10 - 5\sqrt{2})} = \\ & \frac{5\sqrt{10}}{2} - \frac{500\sqrt{5} - 250\sqrt{10}}{50} \\ & \frac{125\sqrt{10}}{50} - \frac{500\sqrt{5} - 250\sqrt{10}}{50} = \\ & \frac{375\sqrt{10} - 500\sqrt{5}}{50} \\ & = \frac{15\sqrt{10} - 20\sqrt{5}}{2} \end{aligned}$$

- 2.) $\triangle ABC$ has vertices A(2,6), B(1,4), and C(3,5). Find the length of the altitude from point A to \overline{BC} .

The altitude from (2,6) to the segment joining (1,4) and (3,5) must have slope -2,

so $\frac{y - 6}{x - 2} = -2$ and $y = \frac{1}{2}x + 3.5$, so $(0.5x + 3.5) - 6 = -2(x - 2)$, so $0.5x - 2.5 = -$

$2x + 4$,

So $2.5x = 6.5$, so $x = 2.6$, and $y = (0.5)(2.6) + 3.5 = 4.8$. The altitude has length

$$\begin{aligned} & \sqrt{(2.6 - 2)^2 + (4.8 - 6)^2} = \sqrt{0.36 + 1.44} = \sqrt{1.8} = \sqrt{\frac{9}{5}} \\ & = \frac{3\sqrt{5}}{5} \end{aligned}$$

3.) X% of Y = Z-30. Y% of Z = 0.025X. Z% of X = 10Y. Find x, y, and z.

$$\frac{x}{100} * y = z - 30, \frac{y}{100} * z = \frac{x}{40}, \frac{z}{100} * x = 10y$$

xy=100z-3000, 40yz=100x, xz=1000y, so $y = \frac{xz}{1000}$, substitute this for y into the two

other equations. $\frac{x}{100} * \frac{xz}{1000} = z - 30$

$$\frac{xz}{10000} * z = \frac{x}{40}$$

The x's cancel from the second equation to get $40z^2=10000$, so $z^2=2500$, so $z=50$.

Then $\frac{50x^2}{10000} = 50 - 30$, so $50x^2=2000000$, $x^2=400000$, and $x=200$. Then y=

$$\frac{2000 * 50}{10000} = 10.$$

4.) In $\triangle KLM$, K has coordinates (1,3), L has coordinates (x,5), and M has coordinates (2,x). Find all possible real values of x such that $\triangle KLM$ is a right triangle.

If there is a right angle at K, then $\frac{5-x}{x-2} = -\frac{1}{\frac{5-3}{x-1}}$, or, so

$-2x+6=x-1$, so $3x=7$, and $x = \frac{7}{3}$. If there is a right angle at L, then

$\frac{5-x}{x-2} = -\frac{1}{\frac{5-3}{x-1}}$, so $\frac{x-5}{x-2} = \frac{x-1}{2}$, and $x^2-3x+2=2x-10$, so $x^2-5x+12=0$. No real

solutions since the discriminant is negative. If there is a right angle at M, then

$\frac{5-x}{x-2} = -\frac{1}{\frac{x-3}{2-1}}$ or $\frac{5-x}{x-2} = \frac{-1}{x-3}$, or $\frac{x-5}{x-2} = \frac{1}{x-3}$, so $x^2-8x+15=x-2$, so x^2-

$$9x+17=0, \text{ so } x = \frac{9 \pm \sqrt{13}}{2}$$

5.) Find all values of d such that $\frac{d+1}{2} - \frac{d+3}{4} - \frac{d+5}{6} = \frac{25}{d}$.

$$\frac{d+1}{2} - \frac{d+3}{4} - \frac{d+5}{6} = \frac{25}{d}$$

Mult.by $12d$

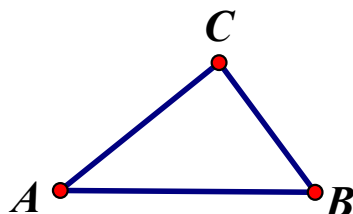
$$6d(d+1) - 3d(d+3) - 2d(d+5) = 300$$

$$6d^2 + 6d - 3d^2 - 9d - 2d^2 - 10d = 300$$

$$d^2 - 13d - 300 = 0$$

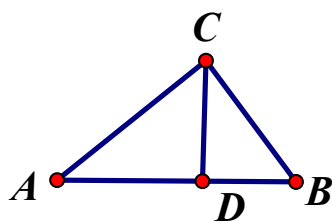
$$(d - 25)(d + 12) = 0$$

$$d=25 \text{ or } d=-12$$



6.) Right triangle ABC above has a right angle at C. If $\tan(\angle ABC)=2$ and $AB=10$, what is the area of the triangle?

Draw the altitude from angle C to segment AB to meet at D.



$$\tan(\angle ABC) = 2,$$

$$\sin(\angle ABC) = 2 \cos(\angle ABC) = 2 \sin(\angle BAC)$$

$$\sin(\angle ABC) = \frac{CD}{BC}, \sin(\angle BAC) = \frac{CD}{AC}$$

$$BC \sin(\angle ABC) = AC \sin(\angle BAC)$$

$$BC * 2 \sin(\angle BAC) = AC \sin(\angle BAC)$$

$$AC = 2 * BC$$

$$\sqrt{(BC)^2 + (2 * BC)^2} = 10$$

$$5 * (BC)^2 = 100, BC = \sqrt{20}, AC = 2\sqrt{20}.$$

$$\text{If } \tan(\angle ABC)=2, \text{ then } \sin(\angle ABC) = \frac{2}{\sqrt{5}}. \quad DC=BC \sin(\angle ABC) = \sqrt{20} * \frac{2}{\sqrt{5}} = 4,$$

So the area is $(1/2)*10*4 = 20$.